

EXPERIMENTAL VERIFICATION OF SHEAR WALL MODELING USING FINITE ELEMENT ANALYSIS

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1. SUMMARY

A three – storey building structure from elastic material is modeled using Lexan and its behavior under an external static loading is examined. Using the photoelastic method we inspect the stresses at shear walls and the experimental results are compared with the corresponding results of a computational model by finite element method application which was developed to describe the problem.

2. INTRODUCTION

The experimental model is a three storey (level) 3D building, having three flat shear walls and one of a “U” – shape, used for the elevator, at every level. The framework plan of the typical level is shown in Fig. 1. Point “A” represents the point in which the external loading is applying. Points “B” and “C” at levels 3 and 2, respectively are the exact points where the displacement is measured.

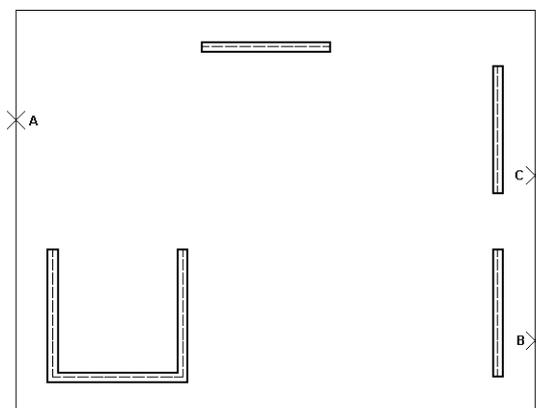


Figure 1: Framework plan of the typical level

Photoelastic analysis is widely used for problems in which stress or strain information is required for extended regions of the structure. It provides quantitative evidence of highly stressed areas and peak stresses at surface and interior points of the structure.

3. FINITE ELEMENT ANALYSIS METHOD

For the finite element analysis of the building we used a four node quadrilateral thin flat shell element, which has six degrees of freedom (dof) per node. The sixth dof is obtained by combining a membrane element with a normal rotation θ_z , the so-called the drilling degree of freedom, and a discrete Kirchhof plate element. The drilling dof is introduced via the variational formulation. The variational formulation employs enforcement of equality of the independent rotation field and skew-symmetric part of the displacement gradient.

In small displacement models of flat shell elements, the effects of membrane and bending strain are not coupled in the energy expression within the elements. Coupling occurs only on the interelement boundary. Therefore, we consider a flat shell element as combination of a plane stress element and a plate bending element. In the combined element subject to membrane and bending actions, the displacements prescribed for 'in-plane' forces do not affect the bending deformations, and vice versa.

The drilling degree of freedom may be physically interpreted as a true rotation of the vertex bisecting the angle between adjacent edges of the finite element. A schematic of the angle bisector and associated partial derivatives in element displacements is shown in Fig. 2.

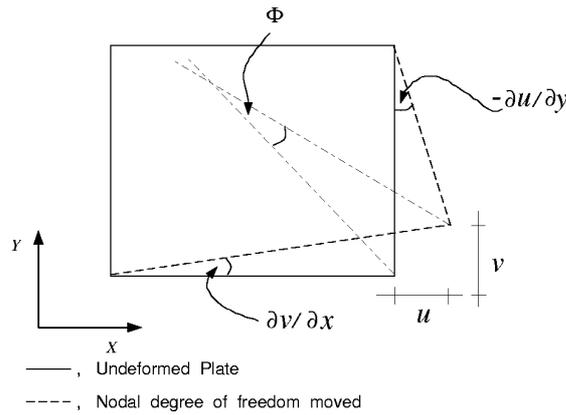


Figure 2: Physical interpretation of the drilling degree of freedom

The drilling degree of freedom is defined as:

$$\Phi = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \quad (1)$$

Flat shell finite elements may be formulated through the use of a variational formulation that includes an independent rotation field for the drilling degree of freedom. The variational formulation is due to Hughes and Brezzi [1, 2]. It employs the skew-symmetric part of the stress tensor as a Lagrange multiplier to enforce the equality of independent rotations with the skew-symmetric part of the displacement gradient. Taylor subsequently combined the variational formulation with an Allman-type interpolation for the displacement field with an independent interpolation field of rotation [3].

The variational formulation suggested by Hughes and Brezzi [1, 3], can be described as

$$\Pi_p(\bar{u}, \bar{\Phi}) = \frac{1}{2} \int_{\Omega} \text{symm}(\nabla \bar{u}) : C \cdot \text{symm}(\nabla \bar{u}) d\Omega + \frac{1}{2} \rho \int_{\Omega} \left| \text{skew}(\nabla \bar{u}) - \bar{\Phi} \right|^2 d\Omega - \int_{\Omega} \bar{u} \cdot f d\Omega \quad (2)$$

where \bar{u} , $\bar{\Phi}$ are trial displacements and rotations of the region Ω , f is the external general forces, and ρ is a penalty. The corresponding variational formulation is:

$$0 = D\Pi_{\rho}\left(u, \Phi\right)\left(\bar{u}, \bar{\Phi}\right) = \int_{\Omega} \text{symm}\left(\nabla \bar{u}\right) \cdot C \cdot \text{symm}\left(\nabla u\right) d\Omega + \rho \int_{\Omega} \left[\text{skew}\left(\nabla \bar{u}\right) - \bar{\Phi} \right]^T \left[\text{skew}\left(\nabla u\right) - \Phi \right] d\Omega - \int_{\Omega} \bar{u} \cdot f d\Omega \quad (3)$$

The first term in the variational equations produces the element stiffness matrix,

$$[K] = \int_{\Omega} [\bar{B}]^T [C] [\bar{B}] d\Omega \quad (4)$$

The plate bending component of the shell element corresponds to the 12 dof discrete Kirchhoff quadrilateral plate element (DKQ), and is derived in detail using the discrete Kirchhoff technique. The DKQ element formulation is based on the discretization of the strain energy. The model neglects the transverse shear strain energy.

4. FINITE ELEMENT ANALYSIS MODEL

The corresponding model is shown in Fig. 3a. It consists of 3496 nodes and 3346 elements. We have tried several mesh sizes and in this paper we present a medium one. The deformed shape of the model is shown in Fig. 3b. The model's elastic isotropic material has a Young modulus of $E = 280000\text{N/cm}^2$, and a Poisson rate of $\nu = 0.36$.

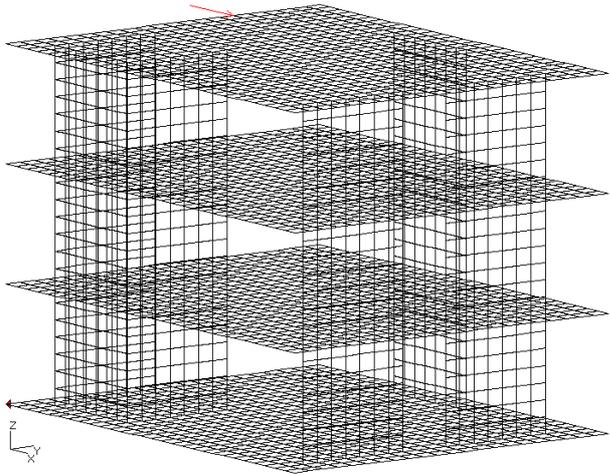
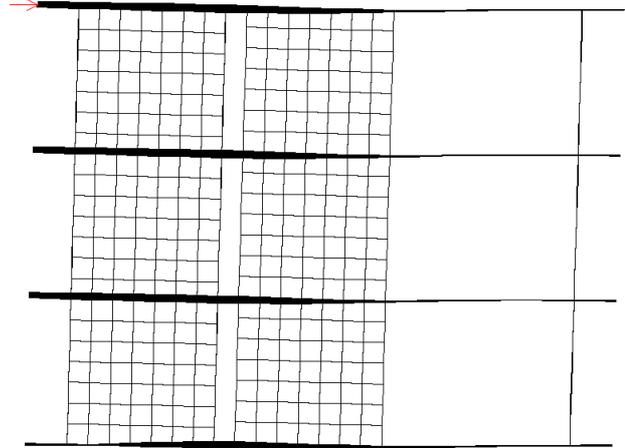


Figure 3a: Finite element analysis model,



3b: Deformed shape of the building

5. EXPERIMENTAL ARRANGEMENT

The specimen was made of Lexan of thickness 6 mm. This material is suitable for both photoelastic and caustic optical method techniques. According to the photoelastic method the specimen is placed between the plates of a circularly polarized field, so that isochromatic fringes patterns can be taken. These fringes give the principal stress difference of an existing stress field. The experimental model is shown in Fig. 4.

The external load is applied at point "A". The starting value is zero and gradually reaches 600N when the joining between the parts of the specimen starts failing. We have also measured the displacements at points "B" and "C" shown in Fig. 1.



Figure 4: Experimental model

6. EXPERIMENTAL RESULTS – COMPARISON

The isochromatic pattern is related to the principal stresses by the stress – optic law:

$$\sigma_1 - \sigma_2 = 2\tau_{max} = \frac{C}{b} \cdot N \quad (5)$$

where σ_1 , σ_2 are algebraically the maximum and minimum principal stresses, respectively, τ_{max} is the maximum shear stress, C is the stress – optic coefficient and N is the relative retardation of rays forming the pattern, also known as isochromatic fringe order.

In terms of the isochromatic pattern, the isochromatic fringe order, N , at a point is specifically defined as the number of fringes that pass through the point during the application of the external loads. The isochromatic pattern of this experimental model is shown in Fig. 5.



Figure 5: Isochromatic pattern

Στο Σχήμα ## μετράμε 4 ισοχρωματικούς κύκλους. Δεδομένου ότι η οπτική σταθερά του Lexan είναι $16.42N/cm$ η διαφορά κυρίων τάσεων προκύπτει ότι είναι $109.47 N/cm^2$. Εξετάσαμε τον πυρήνα και στα τοιχεία υπογείου του τελευταίου ορόφου. Στις παρακάτω εικόνες

that belong to the elevator and the flat shear wall at the top of the framework plan of the typical level (Fig.1). At the next photographs we present the behavior of the flat shear wall at the top of the floor plan, as the value of the external loading is being increased from zero to

600N. The direction of the principal stresses as they are obtained by the finite element analysis are shown in Fig. 7.

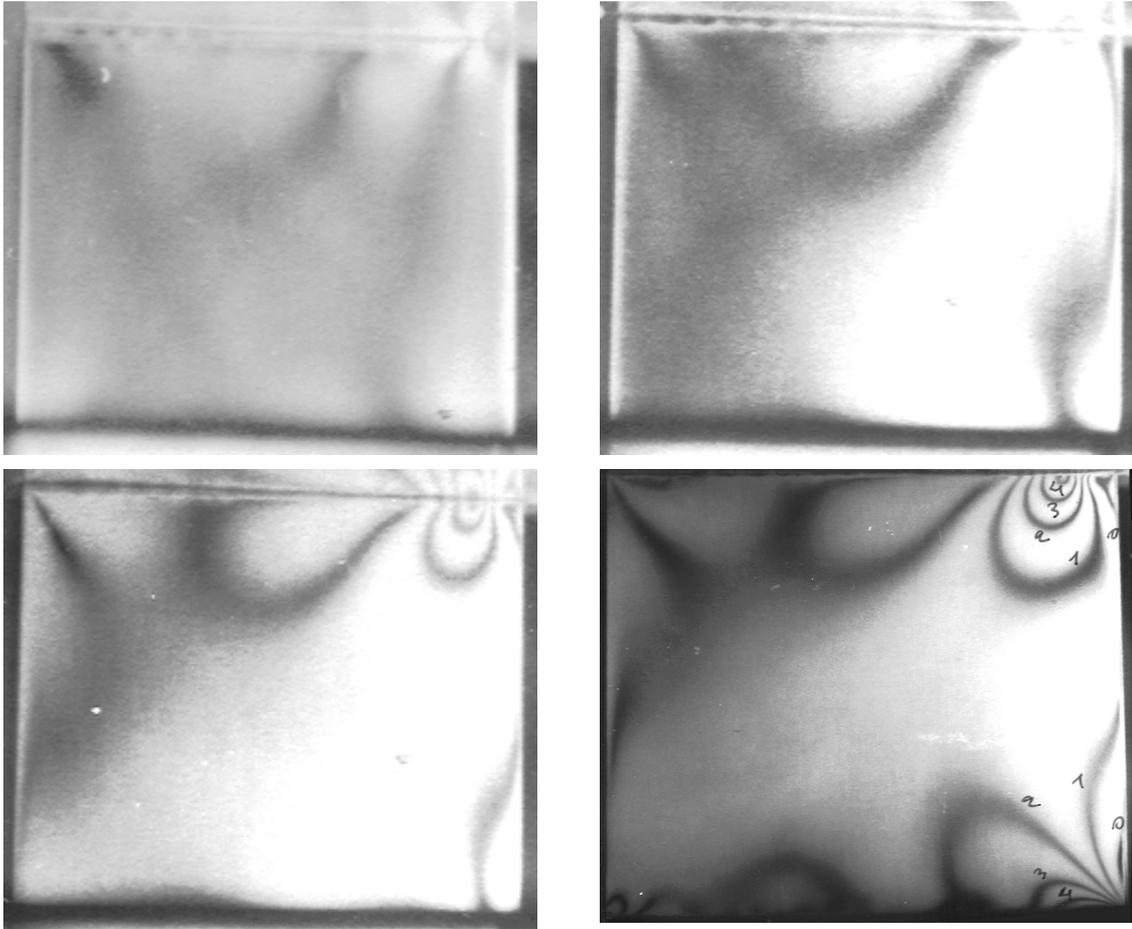


Figure 6: Photoelastic pattern for externally applied load equal to a) 0N, b) 200N, c) 400N and d) 600N

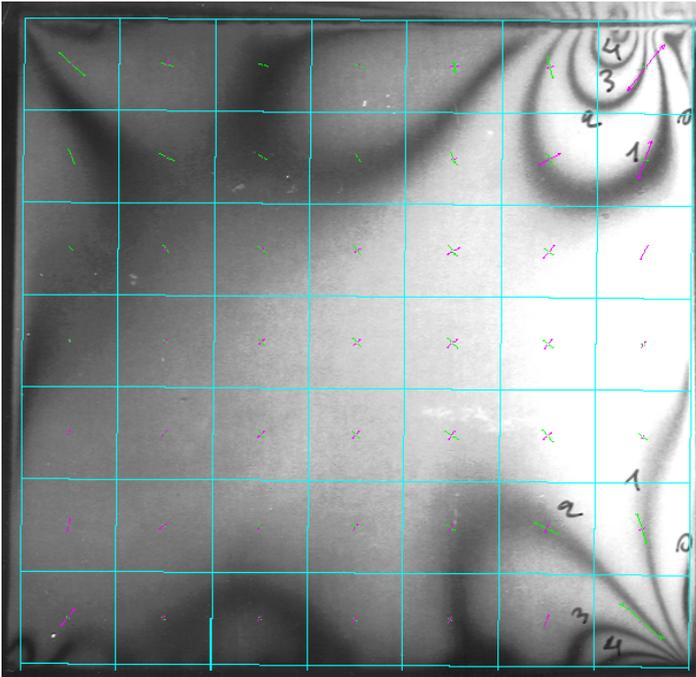


Figure 7: Principal stresses F.E.A.

The comparison between the displacements measured at point B (Level 3), C (Level 2) and those from the finite element analysis are presented at Fig. 8b.

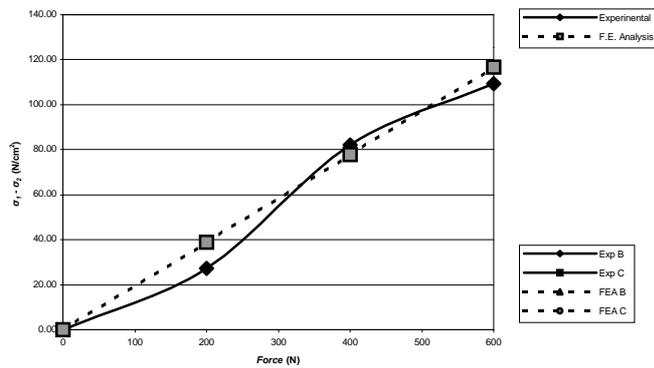
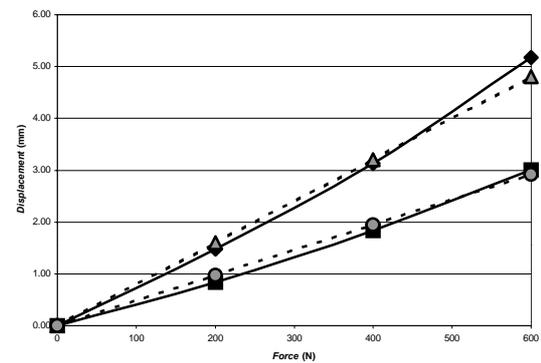


Figure 8a: Experimental principal stresses - F.E.A. ,



8b: Experimental displacements - F.E.A.

7. CONCLUSIONS – FUTURE WORK

The concluding result is that the specific element type can be used to model the experimental specimen in a very satisfactory degree, which is not far enough from a real life building structure. The main assumption that the building's behavior falls into the elastic area will be soon raised as we have already proceeded developing the plastic formulation of the specific element and we shall present it soon.

8. REFERENCES

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